Q Factor Corrections for the source resistance and inductance plus corrections for Lc and Rp of resonating capacitor

Jacques Audet
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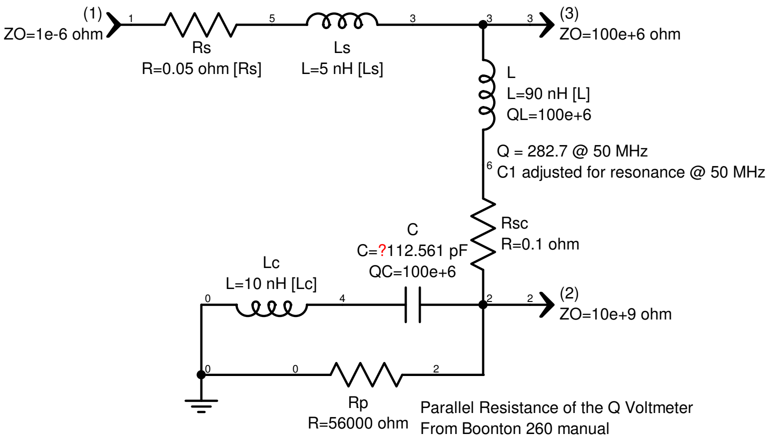


Qm = Q at measured resonant frequency: f (MHz)
Q1 = Q corrected for L only
Qc = Q of capacitor C coming from Rp
Rp = Votmeter resistance (across C)
fo = computed resonant frequency of L and C only (MHz)
L = Inductance of inductor under test: L (µH)
C = Resonant capacitor (pF)
Lc = Residual inductance of Resonant Capacitor C (µH)
Ls = Source inductance (µH)
Xc = Effective Reactance of C at resonance at frequency: f
XL = Reactance of L at resonance at frequency: f
Rs = Source resistance (Ω)
Rsc = ESR of inductor under test (Ω)

$$Xc = \frac{10^6}{2\cdot\pi\cdot f\cdot C} - 2\cdot\pi\cdot f\cdot Lc$$ Xc is the effective reactance of C is in pF at: f

$$Qc = \frac{Rp}{Xc}$$ Qc = factor of the capacitor, including Rp the parallel resistance of the Q Voltmeter

$$XLs = 2\cdot\pi\cdot f\cdot Ls$$ Reactance of source Ls in µH at: f

$$Xc = XL + XLs$$ At resonance: f

$$XL = Xc - XLs$$ At resonance: f

$$XL = \frac{10^6}{2\cdot\pi\cdot f\cdot C} - 2\cdot\pi\cdot f\cdot Ls$$ At resonance: f

$$L = \frac{XL}{2\cdot\pi\cdot f}$$ L of inductor only µH

$$\frac{f}{fo} = \sqrt{\frac{L}{L + Ls + Lc}}$$ The resonant frequencies are inversley proportional to the square root of the inductances

$$fo = \frac{f}{\sqrt{\frac{L}{L + Ls + Lc}}}$$ This is the resonant freq of L and C only

Calculate the Q of L at fo

$$Qm1 = \frac{2\cdot\pi\cdot f\cdot(L + Ls)}{Rs + Rsc}$$

Qm1 = Q of the Inductances without the Capacitor

$$\frac{1}{Qm} = \frac{1}{Qm1} + \frac{1}{Qc}$$

Combining the inductor and capacitor Q's

$$Qm1 = \frac{1}{\frac{1}{Qm} - \frac{1}{Qc}}$$

Solving for Qm1

$$Qm1 = \frac{Qc\cdot Qm}{Qc - Qm}$$

Rearranging

$$Rsc = \frac{2\cdot\pi\cdot f\cdot(L + Ls)}{Qm1} - Rs$$

Solve for Rsc, the ESR of the inductor, assumed to be constant at f and fo

Substitute the expression for Qm1 above into Rsc

$$Rsc = -\frac{Qc\cdot Qm\cdot Rs - 2\cdot\pi\cdot L\cdot Qc\cdot f + 2\cdot\pi\cdot L\cdot Qm\cdot f - 2\cdot\pi\cdot Ls\cdot Qc\cdot f + 2\cdot\pi\cdot Ls\cdot Qm\cdot f}{Qc\cdot Qm}$$

$$Rsc = \frac{2\cdot\pi\cdot L\cdot f}{Qm} - \frac{2\cdot\pi\cdot L\cdot f}{Qc} - Rs - \frac{2\cdot\pi\cdot Ls\cdot f}{Qc} + \frac{2\cdot\pi\cdot Ls\cdot f}{Qm}$$

Rearranging

$$Rsc = \frac{2\cdot\pi\cdot f\cdot(L + Ls)}{Qm} - \frac{2\cdot\pi\cdot f\cdot(L + Ls)}{Qc} - Rs$$

Rearranging

$$Rsc = 2\cdot\pi\cdot f\cdot(L + Ls)\cdot\left(\frac{1}{Qm} - \frac{1}{Qc}\right) - Rs$$

Rearranging

$$Q1 = \frac{2\cdot\pi\cdot fo\cdot L}{Rsc}$$

The corrected Q of inductor = Q1 at fo

Example 1 Lc set to 10 nH The corrected Q = Q1 is Correct

$Qm := 168.87$ $Rs := 0.05$ $Ls := 0.005$ µH $f := 46.294$ MHz $C := 112.561$

$Rp := 56000$ $Lc := 0.010$

$$Xc := \frac{10^6}{2\cdot\pi\cdot f\cdot C} - 2\cdot\pi\cdot f\cdot Lc = 27.634$$

Effective Xc

$$Qc := \frac{Rp}{Xc} = 2.026\times 10^3$$

Q of resonant capacitor C coming from Rp

$$XL := Xc - 2\cdot\pi\cdot f\cdot Ls = 26.18$$

Reactance of coil under test

$$L := \frac{XL}{2\cdot\pi\cdot f} = 0.09$$

Inductance of coil under test

$$fo := \frac{f}{\sqrt{\frac{L}{L + Ls + Lc}}} = 50.003$$

This is the resonant freq fo of L and C only

ESR of inductor under test (Ω)

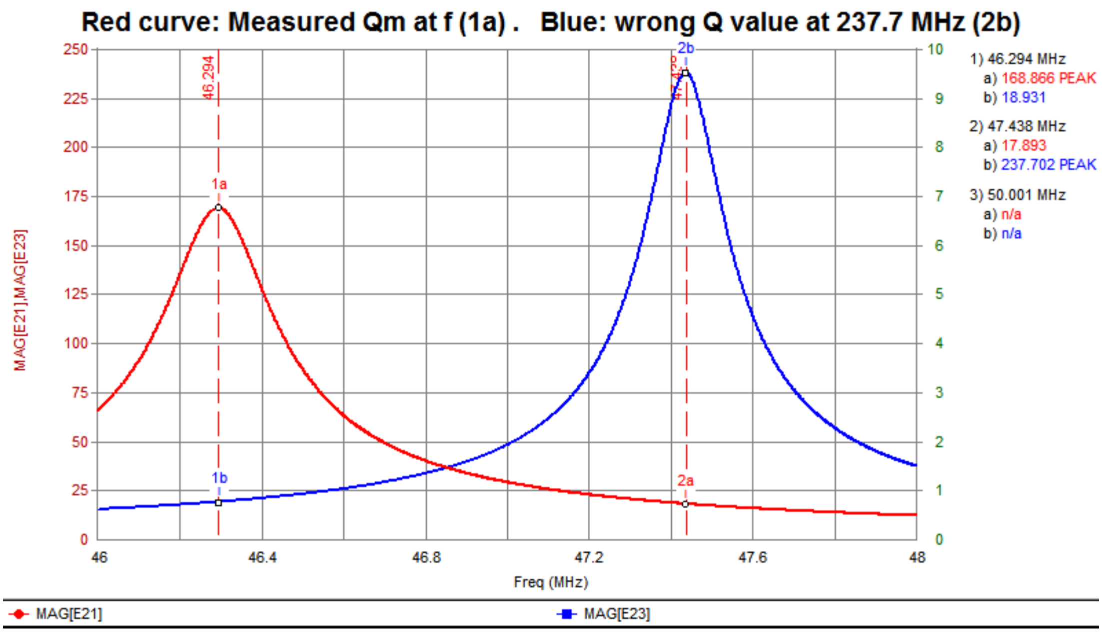
$$Rsc := 2\cdot\pi\cdot f\cdot(L + Ls)\cdot\left(\frac{1}{Qm} - \frac{1}{Qc}\right) - Rs = 0.100004$$

Q @ fo of inductor only, after corrections

$$Q1 := \frac{2\cdot\pi\cdot fo\cdot L}{Rsc} = 282.76$$

Simulations (see top of first page)

Mag[E21] is the voltage (ratio) obtained on a standard Q measurement
Mag[E23] is the voltage ratio: V across Cap / V at source output.



Example 2 Lc set to 0 The corrected Q = Q1 is Correct

$Qm := 176$ $\quad Rs := 0.05$ $\quad Ls := 0.005$ µH $\quad f := 48.669$ MHz $\quad C := 112.561$

$Rp := 56000$ $\quad Lc := 0.000$

$$Xc := \frac{10^6}{2\cdot\pi\cdot f\cdot C} - 2\cdot\pi\cdot f\cdot Lc = 29.052$$

Effective Xc

$$Qc := \frac{Rp}{Xc} = 1.928\times 10^3$$

Q of resonant capacitor C coming from Rp

$$XL := Xc - 2\cdot\pi\cdot f\cdot Ls = 27.523$$

Reactance of coil under test

$$L := \frac{XL}{2\cdot\pi\cdot f} = 0.09001$$

Inductance of coil under test

$$fo := \frac{f}{\sqrt{\frac{L}{L + Ls + Lc}}} = 50.003$$

This is the resonant freq fo of L and C only

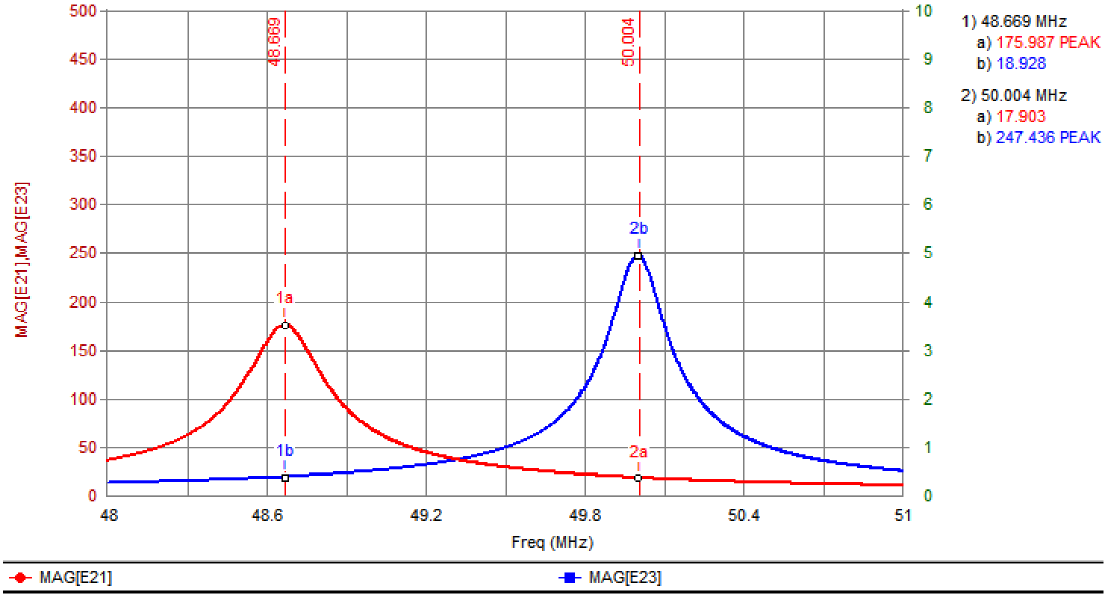
ESR of inductor under test (Ω)

$$Rsc := 2\cdot\pi\cdot f\cdot(L + Ls)\cdot\left(\frac{1}{Qm} - \frac{1}{Qc}\right) - Rs = 0.099998$$

Q @ fo of inductor only, after corrections

$$Q1 := \frac{2\cdot\pi\cdot fo\cdot L}{Rsc} = 282.781$$

Red curve: Measured Qm at f (1a) .   Blue: wrong Q=247.4 @ 50 MHz (2b)



Example 3   Lc and Ls set to 0   The corrected Q = Q1 is correct

$Qm := 172.126$ $Rs := 0.05$ $Ls := 0.000$ μH $f := 50$ MHz $C := 112.561$

$Rp := 56000$ $Lc := 0.000$

$$Xc := \frac{10^6}{2 \cdot \pi \cdot f \cdot C} - 2 \cdot \pi \cdot f \cdot Lc = 28.279$$

Effective Xc

$$Qc := \frac{Rp}{Xc} = 1.98 \times 10^3$$

Q of resonant capacitor C coming from Rp

$$XL := Xc - 2 \cdot \pi \cdot f \cdot Ls = 28.279$$

Reactance of coil under test

$$L := \frac{XL}{2 \cdot \pi \cdot f} = 0.09001$$

Inductance of coil under test

$$fo := \frac{f}{\sqrt{\frac{L}{L + Ls + Lc}}} = 50$$

This is the resonant freq fo of L and C only

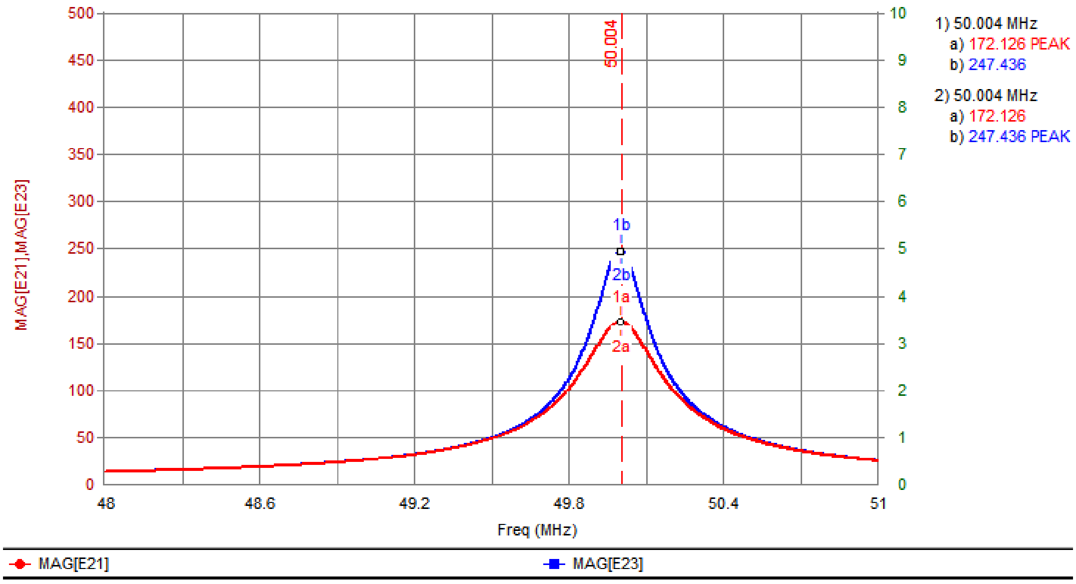
ESR of inductor under test (Ω)

$$Rsc := 2 \cdot \pi \cdot f \cdot (L + Ls) \cdot \left(\frac{1}{Qm} - \frac{1}{Qc} \right) - Rs = 0.100011$$

Q @ fo of inductor only, after corrections

$$Q1 := \frac{2 \cdot \pi \cdot fo \cdot L}{Rsc} = 282.756$$

Red curve: Measured Qm at f (1a) .   Blue: wrong Q=247.4 @ 50 MHz (2b)



Example 4   Lc and Ls set to 0   The corrected Q = Q1 is correct

$Qm := 188.51$ $Rs := 0.05$ $Ls := 0.000$ µH $f := 50$ MHz $C := 112.561$

$Rp := 10^{10}$ $Lc := 0.000$

$$Xc := \frac{10^6}{2\cdot\pi\cdot f\cdot C} - 2\cdot\pi\cdot f\cdot Lc = 28.279$$ Effective Xc

$$Qc := \frac{Rp}{Xc} = 3.536\times 10^8$$ Q of resonant capacitor C coming from Rp

$$XL := Xc - 2\cdot\pi\cdot f\cdot Ls = 28.279$$ Reactance of coil under test

$$L := \frac{XL}{2\cdot\pi\cdot f} = 0.09001$$ Inductance of coil under test

$$fo := \frac{f}{\sqrt{\frac{L}{L + Ls + Lc}}} = 50$$ This is the resonant freq fo of L and C only

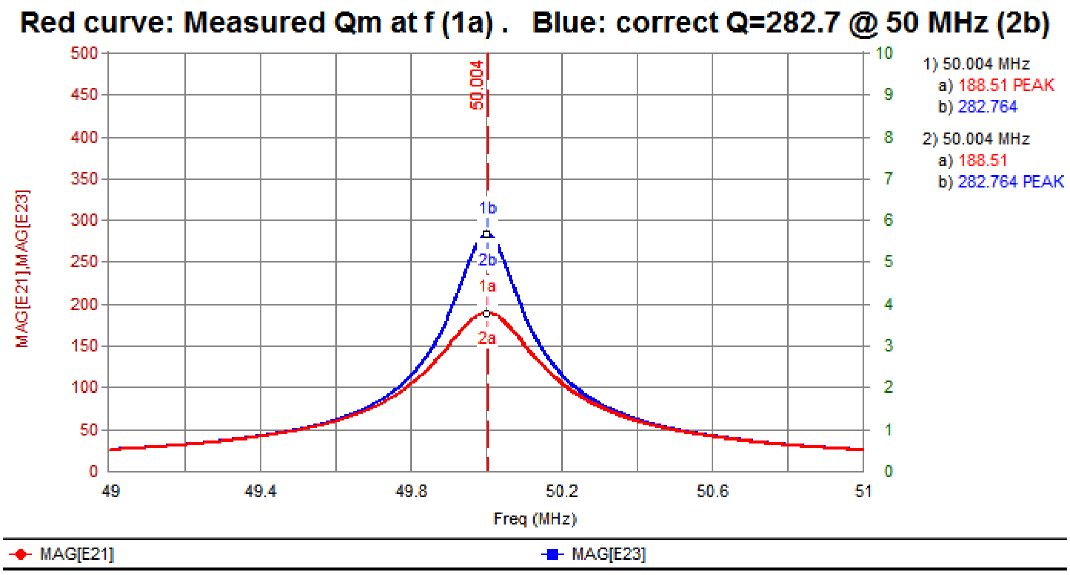
ESR of inductor under test (Ω)

$$Rsc := 2\cdot\pi\cdot f\cdot(L + Ls)\cdot\left(\frac{1}{Qm} - \frac{1}{Qc}\right) - Rs = 0.100013$$

Q @ fo of inductor only, after corrections

$$Q1 := \frac{2\cdot\pi\cdot fo\cdot L}{Rsc} = 282.753$$

Conclusion
Computing Mag[E23] the voltage ratio: V across Cap / V at source output ONLY gives the right results when Rp is very high (negligible).
The Mathcad corrections provide the right Q factor of the coil under test under ALL conditions.



Calculation of Rp (Mohms) vs frequency f (MHz)

$$Rp(f) := 10^{\left(2574.584367+6.708083\cdot f^{-0.15}+26.44179\cdot \ln(f)-2580.071643\cdot f^{0.01}\right)}$$

Ref : Boonton260_Rp Curve.xls.xmcd

Ref. Q Meter Source Sim.wsp