**CALCULATIONS OF L, C in a SERIES CIRCUIT using Leq and Ceq where Leq and Ceq are the apparent values as seen above and below the resonant frequency.**

**See: EXAMPLE OF COMPLETE Rx - L -C CALCULATIONS pages 4 - 6 below as implemented in my Excel file**

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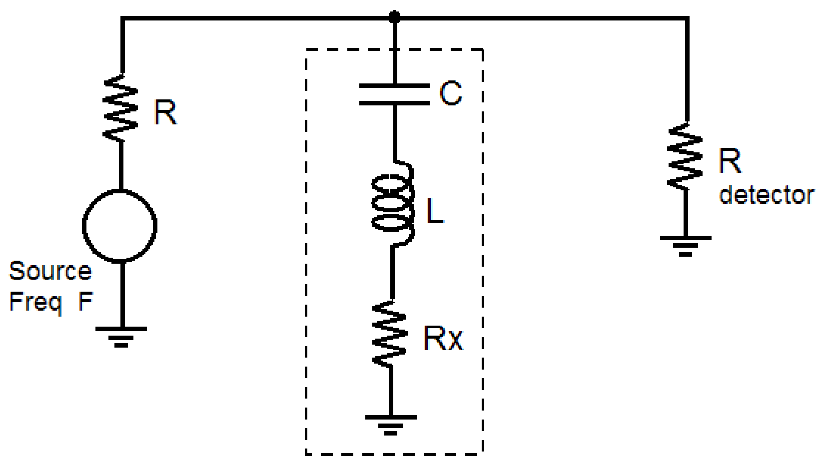


Series R-L-C elements
connected in shunt
in the xmission ckt

Note that the real part of Rx-L-C impedance is always Rx.
It will not influence the calculations of the Leq and Ceq values.

Z_LC_SERIE with Rx

$$Z = j \cdot 2 \cdot \pi \cdot F \cdot L + \frac{1}{j \cdot 2 \cdot \pi \cdot F \cdot C} + Rx$$ general expression

$$Z = Rx + \frac{4 \cdot \pi^2 \cdot C \cdot F^2 \cdot L - 1}{2 \cdot \pi \cdot C \cdot F} \cdot i$$

**If Z is inductive**

$$Leq = \frac{\dfrac{\left(4 \cdot \pi^2 \cdot F^2 \cdot L \cdot C - 1\right)}{2 \cdot \pi \cdot F \cdot C}}{2 \cdot \pi \cdot F}$$

$$Leq = \frac{4 \cdot \pi^2 \cdot F^2 \cdot L \cdot C - 1}{4 \cdot \pi^2 \cdot F^2 \cdot C}$$ eq A

**If Z is capacitive** $Xc = \dfrac{4\cdot\pi^2\cdot C\cdot F^2\cdot L - 1}{2\cdot\pi\cdot C\cdot F} = \dfrac{1}{2\cdot\pi\cdot F\cdot Ceq}$ $\qquad \dfrac{4\cdot\pi^2\cdot C\cdot F^2\cdot L - 1}{2\cdot\pi\cdot C\cdot F} = \dfrac{-1}{2\cdot\pi\cdot F\cdot Ceq}$

Solving for Ceq:

$$Ceq = \frac{C}{1 - 4\cdot\pi^2\cdot C\cdot F^2\cdot L} \qquad \text{eq B}$$

## Example

$F1 := 146\cdot 10^6$ $\qquad L := 100\cdot 10^{-9}$ $\qquad C := 20\cdot 10^{-12}$ $\qquad Rx := 0.1$

Resonant frequency : $Fr := \dfrac{1}{2\cdot\pi\cdot\sqrt{L\cdot C}}$ $\qquad Fr = 1.125\times 10^8$

Since F1 > Fr then Z is inductive

$Z := j\cdot 2\cdot\pi\cdot F1\cdot L + \dfrac{1}{j\cdot 2\cdot\pi\cdot F1\cdot C} + Rx$ $\qquad Z = 0.1 + 37.229i$

$Leq := \dfrac{4\cdot\pi^2\cdot F1^2\cdot L\cdot C - 1}{4\cdot\pi^2\cdot F1^2\cdot C}$ $\qquad Leq = 4.058\times 10^{-8}$

Since F2 < Fr then Z is capacitive

$F2 := 100\cdot 10^6$

$Z := j\cdot 2\cdot\pi\cdot F2\cdot L + \dfrac{1}{j\cdot 2\cdot\pi\cdot F2\cdot C} + Rx$ $\qquad Z = 0.1 - 16.746i$

$Ceq := \dfrac{C}{1 - 4\cdot\pi^2\cdot C\cdot F2^2\cdot L}$ $\qquad Ceq = 9.504\times 10^{-11}$

$$Leq = \frac{4\cdot\pi^2\cdot F1^2\cdot L\cdot C - 1}{4\cdot\pi^2\cdot F1^2\cdot C} \qquad \text{eq A with F replaced by F1}$$

$$Ceq = \frac{C}{1 - 4\cdot\pi^2\cdot C\cdot F2^2\cdot L} \qquad \text{eq B with F replaced by F2}$$

Solve for C in eq. B:

$$C = \frac{Ceq}{4\cdot\pi^2\cdot Ceq\cdot L\cdot F2^2 + 1} \qquad \text{eq C}$$

eq. C into eq. A:

$$Leq = \frac{4\cdot\pi^2\cdot F1^2\cdot L\cdot C - 1}{4\cdot\pi^2\cdot F1^2\cdot C} \qquad \text{eq A}$$

$$Leq = -\frac{4\cdot\pi^2\cdot Ceq\cdot L\cdot F2^2 - 4\cdot\pi^2\cdot Ceq\cdot L\cdot F1^2 + 1}{4\cdot\pi^2\cdot Ceq\cdot F1^2} \qquad \text{eq D}$$

solve for L=

$$L = \frac{4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F1^2 + 1}{4\cdot\pi^2\cdot Ceq\cdot F1^2 - 4\cdot\pi^2\cdot Ceq\cdot F2^2} \qquad \text{eq E}$$

$$L = \frac{4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F1^2 + 1}{4\cdot\pi^2\cdot Ceq\cdot\left(F1^2 - F2^2\right)} \qquad \text{eq E rearranged}$$

Check :

$$L := \frac{4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F1^2 + 1}{4\cdot\pi^2\cdot Ceq\cdot\left(F1^2 - F2^2\right)} \qquad L = 1\times 10^{-7}$$

We get the initial value of L

Sub eq E into eq C:

$$C = \frac{Ceq}{4\cdot\pi^2\cdot Ceq\cdot L\cdot F2^2 + 1} \qquad \text{eq C}$$

$$C = \frac{Ceq\cdot F1^2 - Ceq\cdot F2^2}{4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F1^2\cdot F2^2 + F1^2} \qquad \text{eq F}$$

$$C = \frac{Ceq\cdot\left(F1^2 - F2^2\right)}{F1^2\cdot\left(4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F2^2 + 1\right)} \qquad \text{eq F rearranged}$$

Check :

$$C := \frac{Ceq\cdot\left(F1^2 - F2^2\right)}{F1^2\cdot\left(4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F2^2 + 1\right)} \qquad C = 2\times 10^{-11}$$

We get the initial value of C

## Calculating the L-C series elements from Leq and Ceq in a (R) ohm system after measuring attenuation below and above the resonant frequency

F1 is the highest frequency with a corresponding attenuation: dB1
F2 is the lowest frequency with a corresponding attenuation: dB2

As shown above:

$$L = \frac{4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F1^2 + 1}{4\cdot\pi^2\cdot Ceq\cdot\left(F1^2 - F2^2\right)} \qquad \text{eq E}$$

$$C = \frac{Ceq\cdot\left(F1^2 - F2^2\right)}{F1^2\cdot\left(4\cdot\pi^2\cdot Ceq\cdot Leq\cdot F2^2 + 1\right)} \qquad \text{eq F}$$

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## EXAMPLE OF COMPLETE Rx - L - C CALCULATIONS

CALCULATE Rx from the (negative) attenuation measured in dB at the frequency of minimum transmission: FR
R is the load and source resistance.

**Calculation of Rx** (The reactances of L and C cancel at resonnance)

$MHz := 10^6$ $\quad R := 50$ $\quad dB := -40.5926$ $\quad FR := 150\cdot MHz$

On mesure Rx à cette freq

Calculating the shunt resistance Rx:
See: Calcul_RparV11a.mcd or Calcul_RparV11a.pdf

$$Rx(dB) := \frac{1}{2}\cdot\frac{R\cdot 10^{\frac{dB}{20}}}{\left(1 - 10^{\frac{dB}{20}}\right)} \qquad Rx(dB) = 0.23571$$

$Rx := Rx(dB)$

F1 is the highest freq. corresponding to dB1 (dB)

$F1 := 160\cdot MHz$ $\quad F2 := 149\cdot MHz$

$dB1 := -7.24306$ $\quad dB2 := -25.8895$

**Calculation of Leq et Ceq**

$$\mathrm{att1}(\mathrm{dB}) := 10^{\frac{\mathrm{dB1}}{20}} \qquad \mathrm{att2}(\mathrm{dB}) := 10^{\frac{\mathrm{dB2}}{20}}$$

Attenuations in dB are converted to linear attenuation: att1 and att2

The equivalent Inductance Leq may be calculated:
See: Calcul_Lpar-avecRX-V11a.mcd or Calcul_Lpar-avecRX-V11a.pdf

$$\mathrm{Leq}(\mathrm{dB1}) := \frac{\sqrt{4\cdot \mathrm{att1}(\mathrm{dB1})^2\cdot \mathrm{Rx}^2 + 4\cdot \mathrm{R}\cdot \mathrm{Rx}\cdot \mathrm{att1}(\mathrm{dB1})^2 + \mathrm{R}^2\cdot \mathrm{att1}(\mathrm{dB1})^2 - 4\cdot \mathrm{Rx}^2}}{4\cdot \pi\cdot \mathrm{F1}\cdot \sqrt{1 - \mathrm{att1}(\mathrm{dB1})^2}}$$

The equivalent Capacitance Ceq may be calculated:
See: Calcul_Cpar-avecRX-V11a.mcd or Calcul_Cpar-avecRX-V11a.pdf

$$\mathrm{Ceq}(\mathrm{dB2}) := \frac{\sqrt{1 - \mathrm{att2}(\mathrm{dB2})^2}}{\pi\cdot \mathrm{F2}\cdot \sqrt{4\cdot \mathrm{att2}(\mathrm{dB2})^2\cdot \mathrm{Rx}^2 + 4\cdot \mathrm{R}\cdot \mathrm{Rx}\cdot \mathrm{att2}(\mathrm{dB2})^2 + \mathrm{R}^2\cdot \mathrm{att2}(\mathrm{dB2})^2 - 4\cdot \mathrm{Rx}^2}}$$

$$\mathrm{Leq}(\mathrm{dB1}) = 1.21 \times 10^{-8} \qquad \mathrm{Ceq}(\mathrm{dB2}) = 8.473 \times 10^{-10}$$

**Calculating the L-C series elements from Leq and Ceq:**

$$\mathrm{L}(\mathrm{dB1}) := \frac{4\cdot \mathrm{Leq}(\mathrm{dB1})\cdot \pi^2\cdot \mathrm{F1}^2\cdot \mathrm{Ceq}(\mathrm{dB1}) + 1}{4\cdot \pi^2\cdot \mathrm{Ceq}(\mathrm{dB1})\cdot \left(\mathrm{F1}^2 - \mathrm{F2}^2\right)}$$

eq E

$$\mathrm{L}(\mathrm{dB1}) = 9.9945 \times 10^{-8}$$

$$\mathrm{C}(\mathrm{dB2}) := \frac{\mathrm{Ceq}(\mathrm{dB2})\cdot \left(\mathrm{F1}^2 - \mathrm{F2}^2\right)}{4\cdot \mathrm{F2}^2\cdot \mathrm{Leq}(\mathrm{dB2})\cdot \pi^2\cdot \mathrm{F1}^2\cdot \mathrm{Ceq}(\mathrm{dB2}) + \mathrm{F1}^2}$$

eq F

$$\mathrm{C}(\mathrm{dB2}) = 1.12641 \times 10^{-11}$$

$$\mathrm{Fres} := \frac{1}{2\cdot \pi\cdot \sqrt{\mathrm{L}(\mathrm{dB1})\cdot \mathrm{C}(\mathrm{dB2})}} \qquad \frac{\mathrm{Fres}}{\mathrm{MHz}} = 150$$

%Error:

$$\frac{100\cdot (\mathrm{Fres} - \mathrm{FR})}{\mathrm{Fres}} = 3.004 \times 10^{-4}$$

**Calculating the quality factor: Q**

$$\mathrm{Q} := \frac{2\cdot \pi\cdot \mathrm{Fres}\cdot \mathrm{L}(\mathrm{dB1})}{\mathrm{Rx}} \qquad \mathrm{Q} = 399.619$$

SIMULATION WITH SUPERSTAR