Calculation of Lp, Cp, and Rp as a function of Ls, Cs, Rs, Q and Zo

We will calculate Lp, Cp and Rp to obtain the same attenuation as in a series circuit with Ls, Cs, Rs connected in shunt.
Here Lp, Cp, Rp are connected in parallel.

Attenuation from a series resistance Rp = K

Zo = source and detector

$$K = \frac{\dfrac{Zo}{2\cdot Zo+Rp}}{0.5}$$

$$K = \frac{2\cdot Zo}{2.\cdot Zo + Rp}$$

$$Rp = 2.\cdot Zo\cdot\frac{(1-K)}{K}$$

Attenuation from a shunt resistance Rs = K

$$K = \frac{\dfrac{\dfrac{Zo\cdot Rs}{Zo+Rs}}{\dfrac{Zo\cdot Rs}{Zo+Rs}+Zo}}{0.5}$$

$$K = \frac{2\cdot Rs}{2\cdot Rs + Zo}$$

To get the two attenuations identical:

$$\frac{2\cdot Zo}{2.\cdot Zo + Rp} = 2\cdot\frac{Rs}{(2\cdot Rs + Zo)}$$

$$Rp = \frac{Zo^2}{Rs}$$

Value of Rp to obtain the same attenuation as with the series Rs, Ls, Cs connected in shunt.

TWO BASIC NETWORK RELATIONS:

$$Rp1 = \left(Q^2 + 1\right) \cdot Rs$$

$$Xp = Xs \cdot \frac{\left(1 + Q^2\right)}{Q^2}$$

$$s \cdot Lp = s \cdot Ls \cdot \frac{\left(1 + Q^2\right)}{Q^2}$$ Donc : $$Lp = Ls \cdot \frac{\left(1 + Q^2\right)}{Q^2}$$

Note that: Lp*Cp = Ls*Cs

$$\frac{1}{sCp} = \frac{1}{sCs} \cdot \frac{\left(1 + Q^2\right)}{Q^2}$$ Donc : $$Cp = Cs \cdot \frac{Q^2}{\left(1 + Q^2\right)}$$

In a RLC series circuit, we have: Q, Ls Cs Rs

Rp equivalent:

$$Rp1 = \left(Q^2 + 1\right) \cdot Rs$$

Looking for Rp2 giving an attenuation Ks:

$$Rp2 = \frac{Zo^2}{Rs}$$

$$S = \frac{Rp1}{Rp2}$$ Impedance Scaling factor

$$S = \frac{\left(Q^2 + 1\right) \cdot Rs^2}{Zo^2}$$

$$Lp = \frac{Ls}{S} \cdot \frac{\left(1 + Q^2\right)}{Q^2}$$

$$Lp = \frac{Ls}{Rs^2} \cdot \frac{Zo^2}{Q^2}$$

$$Cp = Cs \cdot S \cdot \frac{Q^2}{\left(1 + Q^2\right)}$$

$$Cp = Cs \cdot \frac{Rs^2}{Zo^2} \cdot Q^2$$

Exemple of calculations:

$Ls := 2582.92$ $\quad Cs := 115.74$ $\quad Rs := 8.386$ $\quad Zo := 50$

$$Fr := \frac{1}{2 \cdot \pi \cdot \sqrt{Ls \cdot 10^{-3} \cdot Cs \cdot 10^{-6}}} \qquad Fr = 9.2049814$$

$$Q := \frac{2 \cdot \pi \cdot Fr \cdot Ls \cdot 10^{-3}}{Rs}$$

$$Q = 17.814$$

$$Lp := \frac{Ls}{Rs^2} \cdot \frac{Zo^2}{Q^2} \qquad Lp = 289.35$$

$$Cp := Cs \cdot \frac{Rs^2}{Zo^2} \cdot Q^2 \qquad Cp = 1033.168$$

$$Rp := \frac{Zo^2}{Rs} \qquad Rp = 298.116$$

VERIFICATIONS

$$Zs(f) := Rs + j \cdot 2 \cdot \pi \cdot f \cdot Ls \cdot 10^{-3} + \frac{1}{j \cdot 2 \cdot \pi \cdot f \cdot Cs \cdot 10^{-6}}$$

$$Y(f) := \frac{1}{Rp} + \frac{1}{j \cdot \left(2 \cdot \pi \cdot f \cdot Lp \cdot 10^{-3}\right)} + j \cdot 2 \cdot \pi \cdot f \cdot Cp \cdot 10^{-6}$$

$$Zp(f) := \frac{1}{Y(f)}$$

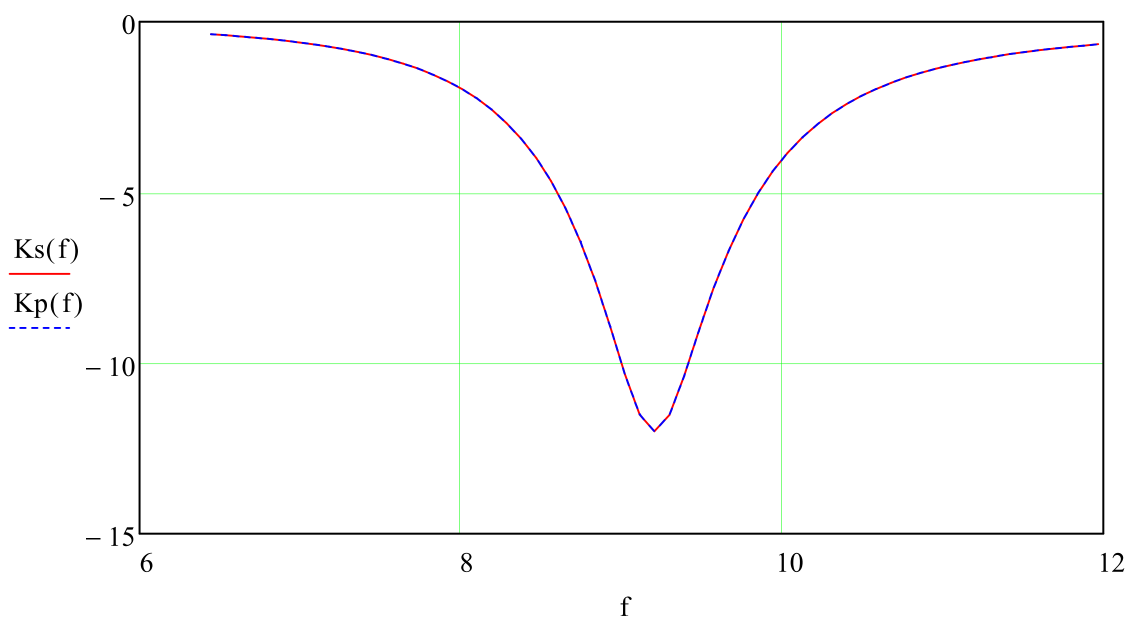
$$Fr = 9.2049814$$

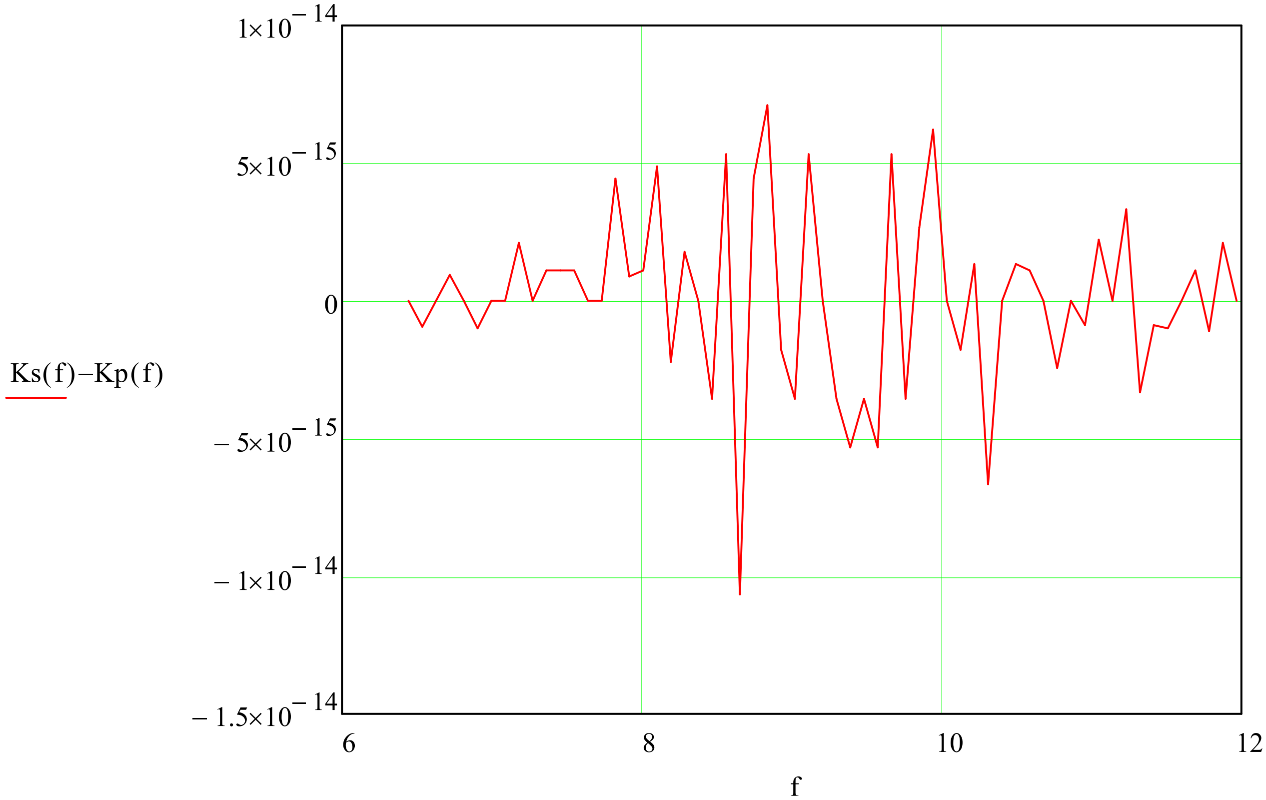
$$Zs(Fr) = 8.386 \qquad Zs(8.9005) = 8.386 - 10.05187911i$$

$$Ks(f) := 20 \cdot \log\left(\left|\frac{2 \cdot Zs(f)}{2 \cdot Zs(f) + Zo}\right|\right) \qquad Ks(Fr) = -12.00019062$$

$$Kp(f) := 20 \cdot \log\left(\left|\frac{2 \cdot Zo}{2. \cdot Zo + Zp(f)}\right|\right) \qquad Kp(Fr) = -12.00019062$$

$$f := 0.7 \cdot Fr, 0.71 \cdot Fr .. 1.3 \cdot Fr$$





1×10^− 14
5×10^− 15
0
− 5×10^− 15
− 1×10^− 14
− 1.5×10^− 14
Ks(f)−Kp(f)
6
8
10
12
f